

The streaming instability: a review

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Abstract. Streaming instabilities are thought to play a fundamental role in both acceleration and propagation of Cosmic Rays (CR) in the Galaxy. While work on this subject dates back to the '70s, important progress has been made in recent years with the discovery of a non-resonant mode of the instability that may provide the large levels of turbulence required to account for particle acceleration up to the highest energies observed in Galactic CRs. I will give a brief overview of our understanding of streaming instabilities and their role in Cosmic Ray physics, following the historical development of studies in this field, from the pioneering works by Skilling and Wentzel in the '70s to the most recent works, both theoretical and numerical, on the non-resonant modes.

Key words. Instabilities – Magnetic Fields – Shock Waves – Acceleration of Particles

1. Introduction

Historically, the interest of the CR physics community for streaming instability was motivated by the problem of explaining CR propagation through the Galaxy. Observations of CRs show that these particles are confined in the Galaxy for a time $T_{\text{conf}} \sim 2 \times 10^7 E_{\text{GeV}}^{-\alpha}$ yr, with $\alpha \sim 0.3 - 0.6$ and E_{GeV} the particle energy in GeV. Most intriguing, an extremely low level of anisotropy is found, $\delta \sim 1.5 \times 10^{-4}$, implying that these particles drift at a velocity $v_D \sim c\delta \sim 50\text{km/s}$. This speed is comparable with the Alfvén velocity in the ionized ISM, v_A , leading to suggest, already in the '70s (see Wentzel (1974) for a review), that isotropization could be provided by wave-particle interactions. The streaming of CRs along magnetic field lines at super-alfvénic speeds generates magnetic turbulence at a wavelength corresponding to the particles' gyroradius (Skilling

1975): such turbulence could then provide efficient scattering of the same particles so as to ensure a small diffusion coefficient and hence small anisotropy and large confinement times in the Galaxy.

Particle streaming is even more important in the acceleration region. CRs up to the so called "knee", namely up to the energy $E_{\text{knee}} \sim 3 \times 10^{15}$ eV where the CR spectrum is observed to steepen, are believed to be of Galactic origin, and Supernova Remnant (SNR) shocks are the best candidate sources based on energetic arguments. The best candidate mechanism is the 1st order Fermi process or Diffusive Shock Acceleration (DSA), with an acceleration rate that scales with the inverse of the diffusion coefficient in the surroundings of the shock: given the finite lifetime of a SNR shock, reaching E_{knee} requires very effective particle scattering, leading to infer a turbulent magnetic field in the shock region much in excess of the average field in the ISM. Such field amplification, of

which we recently acquired observational evidence (Bykov et al. 2011), is thought to be provided by CRs streaming ahead of the shock.

In recent times, the streaming instability has been reconsidered to show that in addition to the well known and long studied resonant (*res*) mode, there is also a non-resonant (*nr*) mode in the dispersion relation, that might show in some situations very large growth rates, much larger than for the *res* one (Bell 2004, B04 hereafter). These *nr* waves lead to magnetic field strengths that could ensure, in principle, acceleration of particles up to the knee. The problem with these modes, in terms of their efficacy for particle diffusion (and hence acceleration), is that they are born as short wavelength modes and some inverse cascading is necessary before they can provide effective particle scattering.

The latter issue can only be addressed by means of numerical studies of the instability, aimed at clarifying its saturation both in terms of strength and dominant wavelength.

2. Wave-particle interaction

Let us consider the interaction between a particle propagating in a magnetic field \mathbf{B}_0 and an Alfvén wave of wavelength λ . Alfvén waves are perpendicular, low frequency waves, with dispersion relation $\omega = kv_A$ where $k = 2\pi/\lambda$ and $v_A = B_0/\sqrt{4\pi n_i m_i}$ (m_i and n_i are the mass and number density of the background ions). When a particle of momentum p and charge e interacts with such a wave, the Lorentz force acting on the particle leads to a change in particle parallel momentum:

$$\Delta p_{\parallel} = e \int_0^{\tau} dt \left(\frac{\mathbf{v} \wedge \mathbf{B}}{c} \right)_{\parallel} \propto \int_0^{\tau} dt (A_+ + A_-) \quad (1)$$

where $A_{\pm} = \cos[(kv_{\parallel} - \omega \pm \Omega)t + \Phi_{\pm}]$, with Φ_{\pm} taking into account the relative phase between particle and wave, $\Omega = eB_0/p$ is the relativistic particle cyclotron frequency and the subscripts \parallel and \perp are with respect to the direction of \mathbf{B}_0 . For positive (negative) values of v_{\parallel} , the integrand in Eq. 1 includes a high frequency term, A_+ (A_-) that averages out over the duration of the interaction $\tau = 2\pi/(kv_{\parallel} - \omega)$,

and a low frequency term, A_- (A_+) that is maximum when the resonance condition, $kv_{\parallel} \approx \Omega$, is satisfied. When this happens, Eq. 1 gives

$$\Delta p_{\parallel} = \pi p_{\perp} (\delta B/B_0) \cos \Phi \quad (2)$$

which can be rewritten in terms of the particle pitch angle θ (angle between the particle propagation direction and \mathbf{B}_0) as $\Delta\theta = -\pi(\delta B/B_0) \cos \Phi$. If we then consider the effect of interactions over a time t much larger than τ , we can define the pitch angle diffusion coefficient:

$$D_{\theta\theta} = \frac{\langle (\Delta\theta)^2 \rangle}{t} = \frac{\pi}{8} \Omega \left(\frac{\delta B}{B_0} \right)^2, \quad (3)$$

and the associated spatial diffusion coefficient

$$D(p) = \frac{c^2}{6D_{\theta\theta}} = \frac{4}{3\pi} \left(\frac{B_0}{\delta B} \right)^2 cr_L \quad (4)$$

where r_L is the particle Larmor radius and $v \sim c$ has been assumed. Isotropy in the wave frame is reached in a time $T_{\text{iso}} \sim D_{\theta\theta}^{-1}$. Therefore the condition that CRs are isotropized in a time $T_{\text{iso}} \ll T_{\text{conf}}$ requires $(\delta B/B_0) \gg 10^{-10} E_{\text{GeV}}^{(1+\alpha)/2}$ at $\lambda \sim 10^{12} E_{\text{GeV}} \text{cm}$.

The most natural way to have turbulence at the right scales is by having CRs injecting it. The growth-rate of CR injected turbulence, γ_w , can be computed following a very simple reasoning that provides a surprisingly accurate answer (see Kulsrud 2004). Let us consider monoenergetic CRs interacting with seed Alfvén waves. Initially, the resonant particle momentum density is $P_{\text{CR}} = n_{\text{CR}}^* m_i \gamma_{\text{CR}} v_D$, with $n_{\text{CR}}^* = n_{\text{CR}}(p > eB_0/c k)$, where k is the wavenumber. After a time $\tau \approx D_{\theta\theta}^{-1}$: $P_{\text{CR}} = n_{\text{CR}}^* m_i \gamma_{\text{CR}} v_A$, and hence $dP_{\text{CR}}/dt = (n_{\text{CR}}^* m_i \gamma_{\text{CR}} (v_D - v_A))/\tau$. This must correspond to the momentum density gained by the waves, which is readily written in terms of γ_w as $dP_w/dt = (1/v_A)(dE_w/dt) = (\gamma_w/v_A)(\delta B^2/8\pi)$, where $v_A = \omega/k$ relates the wave momentum and energy density, P_w and E_w . By equating dP_w/dt and dP_{CR}/dt , and using the expression for $D_{\theta\theta}$ in Eq. 3, we obtain

$$\gamma_w \approx \frac{\pi}{4} \frac{n_{\text{CR}}^*}{n_i} \Omega_0 \frac{v_D - v_A}{v_A}, \quad (5)$$

with $\Omega_0 = eB_0/(m_i c)$ the non-relativistic cyclotron frequency. Eq. 5 implies that super

-alfvénic streaming of particles makes seed Alfvén waves unstable.

While the expression of γ_w in Eq. 5 is an excellent approximation for the growth rate of the *res* mode of the streaming instability, the *nr* mode can only be found through the standard formal theory, which consists of the following steps: take the unperturbed distribution functions of CRs and background plasma; assume that a small amplitude Alfvén wave perturbs them and find the perturbed distributions; compute the resulting currents; use these in Maxwell's equations to derive the linear evolution of waves with time. All of this translates into writing a dispersion relation, that for circularly polarized Alfvén waves reads: $[c^2 k^2 / \omega^2 - 1 - \sum_s \chi_s] = 0$, where the sum is over all the species s and χ_s is the corresponding response. We will write all equations in the frame in which CRs are isotropic.

When the *nr* mode of the CR streaming instability was first highlighted by B04, a much debated topic was the form of the return plasma current \mathbf{J}_{ret} compensating the CR current: one may assume that the charge non-neutrality induced by the streaming CRs is compensated by an equal number of negative charges that are cold and isotropic in the same reference frame (Zweibel 2003); or that a charge imbalance remains in the background plasma and translates into a different drift velocity between the ions and the electrons (Achterberg 1983). In the first scenario ions and electrons in the background plasma have equal density and drift at v_D ; while in the second: $n_e = n_i + n_{\text{CR}}$, $v_i = v_D$ and $v_e = (n_i/n_e)v_D$ in order to ensure charge and current neutrality. It is possible to show (Amato & Blasi 2009, AB09 hereafter) that both assumptions lead to the same dispersion relation to order $\mathcal{O}[(n_{\text{CR}}/n_i)^2]$.

The background plasma response is most easily computed in cold plasma theory, and one obtains (AB09) :

$$\chi_p = -\frac{4\pi e^2 n_i}{\omega^2 m_i} \left[\frac{\omega + kv_D}{\omega + kv_D \pm \Omega_{0i}} + \frac{m_i}{m_e} \frac{\omega + kv_D}{\omega + kv_D \pm \Omega_{0e}} + \frac{n_{\text{CR}} m_i}{n_i m_e} \frac{\omega}{\omega \pm \Omega_{0e}} \right] \quad (6)$$

For the isotropic CRs (Krall & Trivelpiece 1973):

$$\chi_{\text{CR}} = \frac{4\pi^2 e^2}{\omega} \int_0^\infty dp p^2 v \frac{\partial f_{\text{CR}}}{\partial p} \int_{-1}^{+1} d\mu \frac{(1-\mu^2)}{\omega - kv\mu \pm \Omega_i}$$

where μ is the pitch angle cosine, $f_{\text{CR}}(p) = \frac{n_{\text{CR}}}{2} g(p)$ and $g(p)$ is a power-law, normalized so that $\int_{p_0}^{p_{\text{max}}} dp p^2 g(p) = 1$. The integral in μ reduces to

$$-i\pi \int_{-1}^1 d\mu (1-\mu^2) \delta(-kv\mu \pm \Omega_i) + \mathcal{P} \int_{-1}^1 d\mu \frac{(1-\mu^2)}{-kv\mu \pm \Omega_i} \quad (7)$$

The first term in Eq. 7 is the classical resonant part: since $|\mu| \leq 1$, this is non-zero only if $\Omega/kv \leq 1$, or $p \geq p_1 = eB_0/cv$. The second term is the *nr* contribution. In the end one can write:

$$\chi_{\text{CR}} = \frac{c^2 \Omega_0 n_{\text{CR}}}{v_A^2 \omega n_i} (iI_{\text{res}} \mp I_{\text{nr}}) \quad (8)$$

$$I_{\text{res}}(k) = \frac{\pi}{2} \int_{p_1}^\infty dp p p_1 g(p)$$

$$I_{\text{nr}}(k) = -\frac{p_1^3}{2} \int_{\rho_{\text{min}}}^\infty d\rho \rho g(\rho) \ln \left| \frac{1+\rho}{1-\rho} \right| \quad (9)$$

with $\rho = p/p_1$.

The full dispersion relation is obtained by summing χ_{CR} and χ_p , after expansion of the latter for $\omega \ll \Omega_{0i} = -(m_e/m_i)\Omega_{0e} \ll |\Omega_{0e}|$. In the lab frame, namely replacing ω with $\tilde{\omega} - kv_D$ and neglecting $\tilde{\omega}$ with respect to kv_D , one obtains:

$$v_A^2 k^2 = \tilde{\omega}^2 - kv_D \Omega_0 \frac{n_{\text{CR}}}{n_i} (iI_{\text{res}} \pm (1 + I_{\text{nr}})) \quad (10)$$

The lower sign corresponds to right-hand (*rh*) circular polarization (electric field rotation is counter-clockwise): here is where the *nr* term becomes important. The underlying physics is as follows: the background plasma interacts with the (transverse) magnetic field of the waves through a $\mathbf{J}_{\text{ret}} \wedge \mathbf{B}$ term that forces transverse motion of the plasma (the analogous term acting on the CRs has negligible effect due to the high rigidity of these particles); for *rh* polarized waves this causes a stretch of the magnetic field lines that amplifies the perturbation as soon as it is strong enough to overcome the magnetic field line tension. But when does this happen?

3. CR propagation in the Galaxy

In the ISM the CR density is $n_{\text{CR}} \approx 10^{-9} \text{cm}^{-3}$ and $v_D \approx v_A$. This implies that the second term in the

rhs of Eq. 10 is small. The waves are still close to Alfvén waves with $\tilde{\omega} = kv_A + \omega_1$ and $\text{Re}(\omega_1) \ll kv_A$. One then obtains from Eq. 10 and expression for γ_w identical to that in Eq. 5.

In this case, the *lh* and *rh* polarized modes are identical. The *nr* term is irrelevant: confinement of CRs in the Galaxy is determined by competition between growth and damping of the *res* mode alone. Therefore, most of the results already found in the '70s apply. These were showing the difficulty at explaining the isotropy of particles with 100 GeV and larger energies, both in the partially neutral phase of the ISM, where damping is due to ion-neutral collisions, and in the fully ionized phase, where wave growth is limited by non-linear Landau damping. At the same time, it is not easy to explain the scaling with particle energy of the confinement time: this is found to scale as $E^{-\delta}$ with $\delta = 0.3 - 0.6$, while the above mentioned mechanisms would lead to quite larger values of δ : $\delta = 1.7$ and $\delta = 0.85$ respectively (Wentzel 1974). Currently, there are hints, from the combination of observations and theory, that $\delta = 1/3$ is preferable (Blasi & Amato 2011; Ptuskin et al. 2006), making the disagreement even stronger (however see the review by Ptuskin in these proceedings for a more thorough discussion of the topic).

4. CR acceleration at SNR shocks

Let us now move to the problem of getting CRs accelerated up to E_{knee} . If this occurs through DSA at SNR shocks, the particles gain energy each time they cross the shock front, and the gain is $\Delta E/E = (4/3c)(u_1 - u_2)$, with u_1 and u_2 the fluid velocity up- and downstream of the shock respectively. The maximum energy particles can reach is determined by the condition that the acceleration time does not exceed the lifetime of the system: $T_{\text{acc}}(E_{\text{max}}) = T_{\text{age}}$, where $T_{\text{acc}} = 3/(u_1 - u_2) \{(D_1/u_1) + (D_2/u_2)\}$ and D_1 and D_2 are the diffusion coefficient up- and downstream. A large E_{max} can only be achieved for small diffusion coefficients $D(E)$, *i.e.* for efficient scattering. From Eq. 4 one finds that if δB is the same responsible for CR confinement in the Galaxy, then $E_{\text{max}} \sim \text{GeV}$, while if $\delta B \approx B_0$, then $E_{\text{max}} \approx 10^4 - 10^5 \text{GeV}$ (Lagage & Cesarsky 1983), more than one order of magnitude short of the knee.

Studies of shock acceleration in the so-called “non-linear DSA” (NLDSA) regime showed that E_{knee} could be reached at a SNR shock even in the presence of *res* modes alone (Blasi et al. 2007). This result, however, strongly depends on the particle spectrum, which determines the energy depen-

dence of the self-generated diffusion coefficient (see Amato & Blasi (2006) for details) making it flatter than Bohm’s at high energy for spectra flatter than p^{-4} and steeper otherwise. In fact, recent evidence from γ -ray observations points towards spectra that are steeper than p^{-4} : this can still be accommodated within the framework of NLDSA (Caprioli 2011), but reaching E_{knee} by scattering on self-generated *res* waves becomes then very difficult, given that the acceleration time increases more than linearly with energy.

The value of E_{max} , however, depends not only on the slope of $D(p)$, but also on its normalization, and can increase if the turbulent field is larger. This is where the *nr* mode comes into play. Let us now consider the solutions of Eq. 10 in a strongly current driven regime, such as that relevant for shock acceleration.

First of all let us quantify the expression “strongly current driven”. Eq. 10 is made up of three terms with a different dependence on k : the *lhs* is $\propto k^2$, while the term $\propto (1 + I_{\text{nr}})$ is linear with k , and the one $\propto I_{\text{res}}$ does not depend on k . Hence, at sufficiently large k the first term will start to dominate over the *res* and *nr* term. This occurs at $k = k_1$ and k_2 respectively (see AB09 for details). The *nr* mode only exists between k_1 and k_2 if $k_2 > k_1$. In formulae:

$$\frac{4\pi J_{\text{CR}}}{c} \frac{r_{L0}}{B_0} = \frac{U_{\text{CR}}}{U_B} \frac{v_D}{c} > \frac{\pi}{4}, \quad (11)$$

where the two different expressions highlight the same condition in terms of the CR current J_{CR} and in terms of the energy density of CRs (U_{CR}) with respect to that of the unperturbed magnetic field (U_B). When Eq. 11 is satisfied the *lh* and *rh* waves develop a substantially different behavior. As it is clearly seen in Fig. 1, the *nr* mode has a growth rate which is much larger than that of the *res* one (compare the dashed curves in the two panels), and also much larger than the real part of ω (compare the dashed and solid curve in the top panel): the mode is purely growing. The maximum growth rate γ_{max} is obtained at $k \sim k_2/2 = 2\pi J_{\text{CR}}/(cB_0)$, which is large by definition (see Eq. 11), and one finds $\gamma_{\text{max}} \approx v_A k_2$.

This large growth rate induced the hope that the *nr* mode could allow to reach E_{knee} at SNR shocks. However, preliminary questions that need to be answered are: 1) where and when does this mode exist? 2) what is the level of saturation? 3) do these waves, which are in principle too short-wavelength for efficient scattering, undergo an inverse cascade that turns them into more useful turbulence?

The first part of question #1 was addressed by Zweibel & Everett (2010) who showed that Bell’s

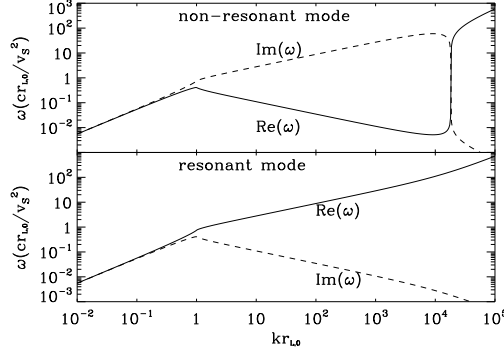


Fig. 1. Real and imaginary part of the frequency as a function of wavenumber for the *rh* (top panel) and *lh* (bottom panel) polarized modes. Frequencies are in units of the inverse of the advection time, $v_s^2/(cr_{L,0})$. The solid (dashed) curve is the real (imaginary) part of ω . The plot is for a shock with $v_s = 10^4$ km/s, $\eta = 0.1$, $B_0 = 1\mu\text{G}$, $n_i = 1\text{cm}^{-3}$.

instability is only relevant, in the form discussed here, for shocks propagating in the cold ISM. For shocks propagating in a hot bubble, a thermally modified version of the instability could only be important for rather large CR densities ($n_{\text{CR}} > 10^{-5}\text{cm}^{-3}$) or rather low magnetic field strength ($B_0 < \text{few } \mu\text{G}$). Even in the case of a SNR expanding in the cold ISM, however, the purely growing *nr* mode exists only during a limited time: as shown by AB09, the mode disappears after $\sim (5-10) \times 10^3$ yr, earlier when B_0 is higher. In conclusion it is potentially important only at the beginning of the Sedov phase (see also Pelletier et al. 2006). On the other hand this is likely when E_{max} is reached.

Finally, let us discuss the problem of non-linear evolution and saturation. Since Bell's original paper, there has been a considerable amount of numerical studies on the subject (Reville et al. 2008; Zirakashvili & Ptuskin 2008; Niemiec et al. 2008; Riquelme & Spitkovsky 2009; Ohira et al. 2009). B04 was predicting saturation to occur at a level where the energy density of the amplified magnetic field equals that of CRs. This implies:

$$\left(\frac{\delta B_{\text{Bell}}}{B_0}\right) \approx \left(\eta \frac{v_D^3}{cv_A^2}\right)^{1/2} \approx 250 \sqrt{\frac{v_4^3 n_1 \eta_{-1}}{B_{\mu\text{G}}^2}}, \quad (12)$$

where v_4 is the shock velocity in units of 10^4km/s , n_1 is the ambient ISM density in units of cm^{-3} , η_{-1} is the fraction of shock ram pressure that is converted into CR energy normalized to 10%, and $B_{\mu\text{G}}$ is the background magnetic field in units of μG . When

one compares Eq. 12 with the level of saturation expected for the *res* mode (see e.g. Amato & Blasi 2006)

$$\left(\frac{\delta B_{\text{res}}}{B_0}\right) \approx \left(\frac{v_D}{v_A} \eta\right)^{1/2} \approx 20 \sqrt{\frac{v_4 n_1^{1/2} \eta_{-1}}{B_{\mu\text{G}}}}, \quad (13)$$

the former turns out to be a factor ~ 10 higher, for typical values of the parameters.

This theoretical level of saturation is in the middle of what numerical studies find under different conditions for the CR current. The work by Riquelme & Spitkovsky (2009) shows that when assuming a constant CR current, the saturation occurs when $v_{A1} \approx v_D$, with v_{A1} the Alfvén velocity in the amplified field: this leads to a saturation δB such that $\delta B/B_0 \approx 4 \times 10^3 (v_4 n_1^{1/2}/B_{\mu\text{G}})$. With mono-energetic CRs, saturation occurs earlier, as soon as the average CR Larmor radius equals the dominant λ of the amplified field. This condition gives saturation at a level: $(\delta B/B_0) \approx 50 (v_4 n_1^{1/3} \eta_{-1}^{1/3}/B_{\mu\text{G}})$, still higher than for the *res* mode, but now only by a factor ~ 2.5 .

An important feature of the non-linear evolution of the instability is that the dominant wavelength increases with time, proportionally to $(\delta B/B_0)^2$ (Riquelme & Spitkovsky 2009). This has very relevant consequences. One of the main objections to the claim that Bell's turbulence can accelerate particles up to E_{knee} has to do with the fact that it is too short-wavelength to produce effective scattering. Early numerical studies were showing subdiffusive behavior at low particle energy E but a E^2 dependence of $D(E)$ at high energy, due to the lack of long λ waves (Reville et al. 2008). In the latter study, however, saturation had not been reached yet and the dominant wave mode was still migrating towards larger λ . Migration can definitely enhance the effectiveness of scattering, but an important caveat is that in reality both the field growth and the migration of λ to useful wavelengths have to occur on timescales shorter than the advection time. This was taken into account, together with a description of scattering on small scale turbulence, by Zirakashvili & Ptuskin (2008) who found that the requirements for particle acceleration up to E_{knee} are rather extreme: a shock velocity of about 40,000 km/s is needed in order to get PeV particles.

5. Conclusions

The *nr* modes of the streaming instability highlighted by B04 generate potentially much larger fields than classically thought. They can easily account for amplified fields observed in SNRs, but

whether they can help us make progress in CR physics is still an open question. Indeed, inclusion of resonant modes alone is perfectly adequate to describe propagation in the Galaxy. Non-resonant modes are expected to become important at SNR shocks during some phases of the SNR evolution, but currently it is not clear that they can provide sufficient scattering for CRs to reach the knee.

In conclusion, while it is interesting that an instability that has been studied for more than 40 years can still reserve surprises, the two main problems to drive its study are still not really solved.

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